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Comparing Groups Using Robust *H* Statistic with Adaptive Trimmed Mean

(Perbandingan Kumpulan Menggunakan *H* Statistik Tegar dan Min Terpangkas Suai)

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ABSTRACT

An alternative robust method for testing the equality of central tendency measures was developed by integrating H *Statistic with adaptive trimmed mean using hinge estimator,* HQ. H *Statistic is known for its ability to control Type I error rates and* HQ *is a robust location estimator. This robust estimator used asymmetric trimming technique, where it trims the tail of the distribution based on the characteristic of that particular distribution. To investigate on the performance (i.e. robustness) of the procedure, some variables were manipulated to create conditions which are known to highlight its strengths and weaknesses. Bootstrap method was used to test the hypothesis. The integration seemed to produce promising robust procedure that is capable of addressing the problem of violations to the assumptions. About 20% trimming is the appropriate amount of trimming for the procedure, where this amount is found to be robust in most conditions. This procedure was also proven to be robust as compared to the parametric (*ANOVA*) and non-parametric (Kruskal-Wallis) methods.*

Keywords: Asymmetric trimmed mean; bootstrap; H *Statistic; hinge estimator; robust statistics; Type I error rates*

ABSTRAK

Kaedah alternatif yang teguh bagi menguji persamaan sukatan kecenderungan memusat telah dibentuk dengan mengintegrasikan H *Statistik dengan min terpangkas suai menggunakan penganggar engsel,* HQ. H *Statistik dikenali kerana kebolehannya untuk mengawal ralat jenis I dan* HQ *adalah penganggar lokasi yang teguh. Penganggar teguh ini menggunakan teknik pemangkasan asimetri, dengan memangkas hujung taburan berdasarkan ciri taburan tersebut. Bagi menguji prestasi (iaitu keteguhan) prosedur, beberapa pemboleh ubah dimanipulasi untuk melihat kekuatan dan kelemahan prosedur. Kaedah butstrap digunakan untuk menguji hipotesis. Integrasi ini menghasilkan prosedur teguh yang mampu menangani masalah pelanggaran andaian. Nilai pemangkas yang sesuai bagi prosedur ini ialah 20% dan didapati tegar dalam kebanyakan keadaan yang dikaji. Prosedur ini juga telah terbukti teguh berbanding kaedah parametrik (*ANOVA*) dan kaedah tidak berparametrik (Kruskal-Wallis).*

Kata kunci: Butstrap; H *statistik; min terpangkas asimetri; penganggar engsel; ralat jenis I; statistik teguh*

INTRODUCTION

In recent years, there have been extensive studies regarding the test on the equality of central tendency measures in terms of their robustness. However, researchers in the field of social sciences, economics and business for example are still attached to the classical tests which are known because of its capacity to comprehensively describe information in the data. When testing for more than two groups, ANOVA will be the most commonly chosen method. Unaware to most of them, this method will be unreliable and produce misleading results when there is any violation in the assumptions. ANOVA has several assumptions that need to be fulfilled before the procedure can be applied. The main assumptions are such that the data should be normally distributed and variances are homogenous. In real situation, these assumptions are often violated and to obtain the ideal data which satisfy all the assumptions is hardly achieved (Wu 2007). Apparently, using data that are not normally distributed with variances that are heterogeneous can cause inaccuracy in the analysis, which will directly mislead the result. According to Wilcox (2012), researcher who falsely assuming normality distributed data risk obtaining biased tests and relatively high Type II error rates for many pattern of non-normality, especially when variance homogeneity is not satisfied. Moreover, the analysis with non-normal data with heterogeneous variances will run the risk of drawing false conclusion from the analysis.

However, there are several alternatives to recover from the limitations of the classical tests such as by trimming the data, transforming the data, using the distribution-free test (non-parametric) (Zar 1996). A non-parametric procedure such as Kruskal-Wallis offers an alternative solution to overcome this problem. Non-parametric tests do not require parametric assumptions because interval data are converted to rank-ordered data and well-known alternatives for dealing with the problem of non-normality. Nevertheless, the non-parametric procedures are criticized over the following reasons: Non parametric procedures possess less power as compared with the classical tests and need larger sample size to reject a false hypothesis

(Syed Yahya 2005). Yu (2010), in his article noted that non-parametric procedures are unable to estimate the true population as they do not make strong assumptions about the population and as a consequence, researchers could not make inference that the sample statistics are estimates of the population parameters.

Robust statistical procedures have solution to the aforementioned problems. They are often used when the data violate any of the normality and homogeneity assumption or both. Robust statistical procedures are insensitive and stable towards the violations of these assumptions. The procedures also deal effectively with outliers. Inspired by the goodness of robust methods, the objective of this study was to propose a new robust test of central tendency measures as an alternative to the classical in dealing with non-normality and inequality of variances. Our proposed robust method is known as $H\hat{T}_{HQ}$ i.e a modification of H Statistic by integrating adaptive trimmed means as the central tendency measure using hinge estimator, HQ.

THE ADAPTIVE TRIMMED MEAN

There are many types of robust estimators to estimate the central tendency measures. One of the estimators is trimmed mean. Wilcox (2012) stated that trimmed mean is relatively insensitive to outliers. It is computed by removing a proportion of the largest and smallest observations and calculates the mean of the remaining observations (Wilcox 2005).

The idea behind trimmed mean is to use a compromise amount of trimming with the goal of achieving a relatively small standard error under both normal and non-normal distribution. Luh and Guo (1999) acknowledged that trimmed mean has an advantage of having a standard error that is less affected by heavy-tailed distributions or outliers. In particular, the trimmed mean is less sensitive to extreme deviation and the non- normal distribution data.

Trimming are done using either symmetric or asymmetric trimming approach. For symmetric trimming, data are trimmed with equal proportion on both tails. However, when the data are asymmetric, trimming both tails with equal amount is no longer appropriate. In such case, the data need to be trimmed with different amount for each tail since sample with longer tail is likely to contain more outliers.

Unlike the symmetric trimming, the proportion of data to be trimmed in asymmetric trimming need to be determined for each tail before the trimming process is performed based on the characteristic of the data. This approach of trimming is called adaptive trimming (Keselman et al. 2007). Since the data are not always symmetric in nature, researchers should always consider the asymmetric trimming approach. Furthermore, the standard deviation are relatively smaller for the adaptive trimmed mean as compared with the usual trimmed mean with fixed amount for both tails (Reed & Stark 1996).

Keselman et al. (2005) found that tests on the equality of groups performed very well with respect to Type I error control in non-normal heteroscedastic distribution when adopting robust estimators based on asymmetric trimming of the data. Baguio (2008) proposed the adaptive robust estimator for the Weibull distribution and the study showed that the adaptive robust estimator was efficient when dealing with this type of distribution. Keselman et al. (2007) examined nine adaptive methods of trimming and observed that eight of the methods have good control of Type I error depending on the degree of non-normality and variance heterogeneity.

METHOD

H STATISTIC

H Statistic was proposed by Schrader and Hettmansperger (1980) and this statistic is readily adaptable to any measure of location. To understand the H statistic, let $X_{(1)}$, $X_{(2)}$, … …, $X_{(nj)j}$, be an ordered sample of group j where $j = 1, 2, \ldots J$. Then, the H statistic can be defined as:

$$\mathrm{H} = \frac{1}{N}\sum_{j=1}^{J} n_j\left(\hat{\theta}_j - \hat{\theta}.\right)^2, \tag{1}$$

where, the overall sample N is determined by number of sample in each group n_j, as in (2).

$$N = \Sigma_j n_j. \tag{2}$$

The mean of the location measures, $\hat{\theta}.$, is defined as:

$$\hat{\theta}. = \Sigma_j \hat{\theta}_j / J. \tag{3}$$

Schrader and Hettmansperger (1980) indicated that this statistic can integrate with any measure of location. In this study, the adaptive trimmed mean with hinge estimator is used as the location measure in each group ($\hat{\theta}_j$) for H Statistic.

HINGE ESTIMATORS

Hinge estimators were proposed by Reed and Stark (1996). They produced seven hinge estimators which are HQ, HQ_1, HQ_2, HH_1, HH_3, HSK_2 and HSK_5. Among these estimators, Keselman et al. (2007) declared HQ as one of the top three best estimators and known to have good control of Type I error rate. They adopted the work of Hogg (1974) and Reed and Stark (1996) to define adaptive trimmed mean based on the measure of tail length for HQ. The HQ is defined in (4).

$$Q = \frac{\left(U_{(0.05)} - L_{(0.05)}\right)}{\left(U_{(0.5)} - L_{(0.5)}\right)}. \tag{4}$$

The value of Q is used to classify symmetric distribution as light-tailed, medium tailed or heavy-tailed. Values of Q < 2 imply light-tailed distribution, 2.0 < Q

≤ 2.6 is a medium-tailed distribution, 2.6 < Q ≤ 3.2 is a heavy-tailed distribution and Q > 3.2 is a very heavy-tailed distribution. $U_{(0.05)}$ and $L_{(0.05)}$ are the mean of the upper and lower 5% of the order statistics of the combined samples while $U_{(0.5)}$ and $L_{(0.5)}$ are the mean of the upper and lower 50% of the order statistics of the combined samples.

Reed and Stark (1996) defined hinge estimators HQ as:

$$HQ = \frac{UW_Q}{UW_Q + LW_Q}, \tag{5}$$

where, UW_Q and LW_Q are calculated:

$$UW_Q = [\Sigma_j n_j (U_{0.05} - L_{0.05}]/\Sigma_j n_j. \tag{6}$$

and

$$LW_Q = [\Sigma_j n_j (U_{0.5} - L_{0.5}]/\Sigma_j n_j. \tag{7}$$

H STATISTIC WITH HQ $(H\hat{T}_{HQ})$

To insert HQ to the *H* Statistic, firstly, we estimated the proportion to be trimmed from the lower end of the sample, as the following equation:

$$\alpha_l = \alpha[UW_Q/(UW_Q + LW_Q)]. \tag{8}$$

The upper end of the sample is given:

$$\alpha_u = \alpha - \alpha_l, \tag{9}$$

where, α is set as a trimming percentage and in this study we used 10, 15, 20 and 25% of trimming (α = 0.1. 0.15, 0.2, 0.25).

Let $Y_{(1)j} \le Y_{(2)j} \le \ldots \le Y_{(n)j}$ represent the ordered observations associated with the *j*th group. The trimmed mean for each group is calculated based on the following formula:

$$\hat{T}_j = \frac{1}{h}\sum_{i=g_1+1}^{n_j - g_2} Y_i, \tag{10}$$

where,

$$g_1 = [n_j \alpha_u]. \tag{11}$$

$$g_2 = [n_j \alpha_1]. \tag{12}$$

$$h = n_j - g_1 - g_2, \tag{13}$$

(g_1 and g_2 represent the amount of trimming from the upper and lower tail, respectively).

Subsequently, we substitute θ_j in (1) with $\hat{T}_j$ as calculated in (10) to obtain $H\hat{T}_{HQ}$:

$$H\hat{T}_{HQ} = \frac{1}{N}\sum_{j=1}^{J} n_j \left(\hat{T}_j - \hat{T}.\right)^2, \tag{14}$$

where

$$N = \Sigma_j n_j, \tag{15}$$

and

$$\hat{T}. = \sum_j T_j / J, \tag{16}$$

$\hat{T}_j$ is the trimmed mean for group *j* and $\hat{T}.$ is the average trimmed mean for all groups.

VARIABLES MANIPULATED

In order to investigate the robustness of the statistic, we manipulated five variables; percentages of trimming, balance and unbalance sample sizes, types of distribution, degree of variance heterogeneity and nature of pairings.

The purpose was to create various conditions with the procedure based on its ability to control Type I error rates. In investigating the performance of $H\hat{T}_{HQ}$ under various trimming percentages, four amount of trimming were assigned namely, α = 0.1, α = 0.15, α = 0.2, α = 0.25. Later in the study, the best trimming for the procedure will be recommended. Comparison was done on four groups (J = 4) of completely randomized design for balanced and unbalanced sample sizes (n_j). Previous research by Othman et al. (2004) using total sample sizes of 70 and 90, respectively, produced Type I error rates close to the nominal value of α = 0.05. Therefore, it can be inferred that total sample size of any value within the 70 and 90 range should produce reasonably good Type I error rates. For the convenience of comparison, the total sample sizes were kept constant at 80. The distribution of the balanced sample sizes among the four groups were $n_1 = n_2 = n_3 = n_4 = 20$. While for the unbalanced sample sizes, the groups with different number of observations were distributed as $n_1 = 10$, $n_2 = 15$, $n_3 = 25$, $n_4 = 30$.

In order to examine the effect of non-normality on the performance of the $H\hat{T}_{HQ}$, three types of distributions had been chosen; standard normal distribution, *g*-and-*h* distribution with g=0.5 and h=0, and *g*-and-*h* distribution with g=0.5 and h=0.5. The standard normal distribution represents distribution with zero skewness, while *g*-and-*h* distribution with g=0.5 and h=0 represents skewed normal-tailed and g=0.5 and h=0.5 represents skewed heavy-tailed. The other assumption that is often violated is the homogeneity of the variances. In order to investigate the effect of variance heterogeneity on Type I error rates, we assigned three degree of variances i.e. equal variances, mild and extreme degree of heterogeneity. Equal variances were set at 1:1:1:1 and for mild and extreme degree of heterogeneity, the unequal variances were assigned at 4:8:16:36 and 1:1:1:36, respectively. When unequal variances were paired with unequal sample sizes, two types of pairings were formed, i.e. negative and positive pairings. A positive pairing involved the pairing of the largest number of group observations with the largest group variance and the smallest group of observations with the smallest group variance. As for negative pairing, the group with the largest number of observations was paired with the smallest group variance, while the smallest group of observations was paired with the largest group variance.

In order to generate data that follow the above conditions, we simulated the data using the SAS/IML version 9.1. The study on distributional shape required the simulation of data according to the type of distribution. The data generation procedure using SAS/IML for $H\hat{T}_{HQ}$ procedure is explained as follows:

NORMAL DISTRIBUTION

Standard normal distribution used straight forward usage of SAS generator RANNOR with mean 0 and standard deviation 1.

For the $g = h = 0.5$ distribution, we generated standard normal distribution as in (a) and transform the standard normal distribution to random variables via (17):

$$Y_{ij} = \frac{\exp(gZ_{ij}) - 1}{g} \exp\left(hZ_{ij}^2 / 2\right). \quad (17)$$

The parameter g controls the amount of skewness, while parameter h controls the kurtosis.

Next, for the $g = 0.5$, $h = 0$ distribution, we modified (17) such that:

$$Y_{ij} = \frac{\exp(gZ_{ij}) - 1}{g}. \quad (18)$$

Since the sampling distribution of H statistic is unknown, bootstrap method was used to test the hypothesis. In order to calculate the Type I error rate, 5000 datasets were generated and each data set was bootstrapped 599 times. In evaluating the robustness of each procedure, we adopt Bradley's (1978) liberal criterion of robustness. According to this criterion, in order for a procedure to be considered robust, its empirical rate of Type I error ($\acute{\alpha}$) must be within $0.5\alpha \le \acute{\alpha} \le 1.5\alpha$. Therefore, for the 0.05 level of significance used in this study, the procedure would be considered robust in a particular condition if its empirical rate of Type I error falls within $0.025 \le \acute{\alpha} \le 0.075$. The proposed procedure was then compared with the parametric and non-parametric procedures represented by ANOVA and Kruskal Wallis, respectively and the results are shown in the following section.

RESULTS

Altogether, a total of 108 conditions were investigated from 6 procedures including ANOVA and Kruskall Wallis. The other 4 procedures were $H\hat{T}_{HQ}$ with different trimming percentages. Each procedure consists of 18 conditions. The results are displayed in Tables 1 – 3. For $H\hat{T}_{HQ}$, 58.3% of the conditions are robust for 10% trimming, 79.2% are robust for both 15% and 25% trimmings, while 20% trimming gained the highest percentage with 87.5%.

Type I error rates for balanced design with homogenous variances are shown in Table 1. In the first row, which represent the ideal condition, the Type I error rates for all the procedures fall between the liberal criterion of robustness with values ranging from 0.0498 to 0.0608. Even for the skewed normal-tail data on the second row, the Type I error rates fall within the robust interval. This signifies that under zero and mild skewness, all procedures including the parametric and non-parametric are robust. However, as the tail becomes heavier, the results of $H\hat{T}_{HQ}$ procedure become more conservative especially at 15 and 25% trimming where the Type I error rates are observed to be below the lower limit of the interval. For this condition, $H\hat{T}_{HQ}$ with 20% trimming, ANOVA and Kruskal Wallis maintain to be robust.

Table 2 shows the results for unbalanced design with homogeneous variances across distributions. Similar with the results from Table 1, the 20% trimming, ANOVA and Kruskal Wallis are proven to be robust in all conditions, meanwhile for 10% trimming, the Type I error rate becomes liberal even when the distribution is normal. On the other hand, at 15 and 25% trimming, the rates become conservative with values less than 0.025.

The performance of the procedures under unbalanced design with heterogeneous variances is presented in Table 3. There is an additional of two extra columns which depict the different degrees of variance heterogeneity and different pairings of group variances and sample sizes. These types of data are common in real life when almost all the assumptions of classical tests are violated (Wu 2007). As could be observed, both ANOVA and Kruskal Wallis become liberal with Type I error rates ranging from 0.1448 to 0.3546 and 0.0794 to 0.1158, respectively, when dealing with negative pairings conditions. The results worsen when the tails become heavier. ANOVA is the worst procedure under extreme conditions (skewed heavy tail). This procedure fails to control Type I error with only 5 out of 12 conditions are proven robust. Meanwhile, the $H\hat{T}_{HQ}$ using

TABLE 1. Type I error rates for equal sample size, N = 80 (20,20,20,20) with homogenous variances, (1,1,1,1)

Distributions	10%	15%	20%	25%	ANOVA	Kruskal-Wallis
Normal	0.06080	0.05440	0.05080	0.05060	0.0518	0.0498
Skewed normal-tailed	0.05700	0.61800	0.03740	0.03640	0.055	0.0498
Skewed heavy-tailed	0.01800	0.02240	0.03680	0.01580	0.029	0.0498

TABLE 2. Type I error rates for unequal sample sizes, N = 80 (10,15,25,30) with homogenous variances, (1,1,1,1)

Distributions	10%	15%	20%	25%	ANOVA	Kruskal-Wallis
Normal	0.0756	0.0654	0.0562	0.0568	0.0504	0.0478
Skewed normal-tailed	0.0672	0.0604	0.0524	0.0440	0.0512	0.0478
Skewed heavy-tailed	0.0304	0.0234	0.0254	0.0202	0.0416	0.0478

TABLE 3. Type I error rates for unequal sample sizes, N = 80 (10,15,25,30) with heterogenous variances, (4,8,16,36) (1,1,1,36)

Distributions	Variances	Pairings	10%	15%	20%	25%	ANOVA	Kruskal-Wallis
Normal	4,8,16,36	Positive	0.0644	0.0686	0.0590	0.0548	0.029	0.0338
	36,16,8,4	Negative	0.0828	0.0654	0.0612	0.0620	0.1662	0.0914
	1,1,1,36	Positive	0.0688	0.0738	0.0662	0.0636	0.0336	0.0448
	36,1,1,1	Negative	0.0936	0.0780	0.0786	0.0750	0.285	0.1158
Skewed normal-tailed	4,8,16,36	Positive	0.0648	0.0636	0.0492	0.0378	0.0308	0.0354
	36,16,8,4	Negative	0.0824	0.0686	0.0688	0.0544	0.1684	0.094
	1,1,1,36	Positive	0.0766	0.0682	0.0616	0.0646	0.0442	0.0462
	36,1,1,1	Negative	0.1152	0.0886	0.0864	0.0872	0.309	0.1146
Skewed heavy-tailed	4,8,16,36	Positive	0.0254	0.0304	0.0312	0.0238	0.0482	0.038
	36,16,8,4	Negative	0.0490	0.0342	0.0344	0.0298	0.1478	0.0794
	1,1,1,36	Positive	0.0510	0.0576	0.0648	0.0570	0.1448	0.0442
	36,1,1,1	Negative	0.1440	0.0772	0.0778	0.0644	0.3546	0.1022

10% trimming also does not perform well as compared with the other trimming percentages. The 15 and 20% trimming show similar pattern under extreme variance heterogeneity, 36,1,1,1 for negative cases. The three conditions produced liberal Type I error rates even when sampling from normal distribution. Under the influence of unbalanced design with heterogeneous variances, 25% trimming produces the best result when 10 out of 12 conditions are identified to be robust.

In general, among all the trimming percentages investigated, the 20% trimming is the appropriate amount of trimming for $H\hat{T}_{HQ}$ procedure, where this amount is found to be robust in most conditions (15 out of 20). The next in sequence are 25%, 15% and 10% trimmings with 14, 11 and 10 of the conditions are robust, respectively.

CONCLUSION

From the overall results, we observe that the proposed procedure $H\hat{T}_{HQ}$, has good control of Type I error. As shown in the analysis, most of the empirical Type I error rates of the procedure fall within Bradley's liberal criterion of robustness interval i.e. between 0.025 and 0.075, as compared with ANOVA and Kruskal Wallis. The $H\hat{T}_{HQ}$ procedure is on par with ANOVA and Kruskal Wallis under ideal condition. However, under extreme condition, $H\hat{T}_{HQ}$ was proven to be extremely better than ANOVA and Kruskal Wallis. This indicates that under severe violation of assumptions, parametric and non-parametric procedures are unable to control Type I error rates. The inflation of Type I error rates will cause the null hypothesis to be spuriously rejected.

In order to avoid from drawing false conclusion, we suggest the 20% trimming of $H\hat{T}_{HQ}$ procedure as this procedure is as good as the parametric and non-parametric procedures when the assumptions are satisfied and maintains its good performance in controlling Type I error (robust) even under severe violation of assumptions.

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